

## Beispielaufgaben zu Gamma-Matrizen

Wir zeigen einige Beziehungen, ohne auf eine Darstellung der  $\gamma^{\mu}$  zurückzugreifen.

- Erinnerung:
- Clifford-Algebra  $\{\gamma^{\mu}, \gamma^{\nu}\} = 2\eta^{\mu\nu} \mathbb{1}$ ,
  - $\not{p} = \gamma^{\mu} p_{\mu} = \gamma_{\mu} p^{\mu}$ ,
  - $p \cdot q = p^{\mu} q_{\mu} = p_{\mu} q^{\mu}$ ,
  - Indexziehen mit  $\eta$ :  $p^{\mu} = \eta^{\mu\nu} p_{\nu}$ .

$$\begin{aligned} \text{a) } \underline{\gamma^{\mu} \gamma_{\mu}} &= \gamma^{\mu} \gamma^{\nu} \eta_{\mu\nu} = \frac{1}{2} (\gamma^{\mu} \gamma^{\nu} \eta_{\mu\nu} + \gamma^{\mu} \gamma^{\nu} \eta_{\mu\nu}) \\ &= \frac{1}{2} (\gamma^{\mu} \gamma^{\nu} + \gamma^{\nu} \gamma^{\mu}) \eta_{\mu\nu} \\ &= \frac{1}{2} \{\gamma^{\mu}, \gamma^{\nu}\} \eta_{\mu\nu} \\ &= \mathbb{1} \eta^{\mu\nu} \eta_{\mu\nu} = \underline{4 \cdot \mathbb{1}} \end{aligned}$$

Umbenennung  $\mu \rightarrow \nu$   
 $= \gamma^{\nu} \gamma^{\mu} \eta_{\nu\mu} = \gamma^{\nu} \gamma^{\mu} \eta_{\mu\nu}$   
 $\eta$  symmetrisch

$$\begin{aligned} \text{b) } \underline{\gamma^{\mu} \not{p} \gamma_{\mu}} &= \gamma^{\mu} \gamma^{\nu} p_{\nu} \gamma_{\mu} = \gamma^{\mu} \gamma^{\nu} \gamma_{\mu} p_{\nu} = \gamma^{\mu} \gamma^{\nu} \gamma^{\sigma} \eta_{\mu\sigma} p_{\nu} \\ &= \underbrace{2\eta^{\nu\sigma} \gamma^{\mu} \eta_{\mu\sigma}}_{\gamma^{\nu}} p_{\nu} - \gamma^{\mu} \gamma^{\sigma} \gamma^{\nu} \eta_{\mu\sigma} p_{\nu} \\ &= 2\not{p} - \underbrace{\gamma_{\sigma} \gamma^{\sigma}}_{= \gamma^{\sigma} \gamma_{\sigma} = 4 \cdot \mathbb{1}} \not{p} = \underline{-2\not{p}} \end{aligned}$$

$= 2\eta^{\nu\sigma} \mathbb{1} - \gamma^{\sigma} \gamma^{\nu}$

$$\begin{aligned}
c) \quad \underline{\gamma^\mu \not{p} \not{q} \gamma_\mu} &= \gamma^\mu \not{p} \gamma^\nu q_\nu \gamma_\mu = \gamma^\mu \not{p} \gamma^\nu \gamma_\mu q_\nu \\
&= \gamma^\mu \not{p} \underbrace{\gamma^\nu \gamma^\mu}_{= 2\eta^{\nu\mu} \mathbb{1} - \gamma^\mu \gamma^\nu} q_\nu \\
&= 2\eta^{\nu\mu} \overset{2.}{\gamma^\mu} \not{p} q_\nu \underbrace{\eta_{\mu\nu}}_1 - \gamma^\mu \not{p} \gamma^\mu \overset{q}{\gamma^\nu} q_\nu \\
&= 2\gamma^\nu \not{p} q_\nu - \underbrace{\gamma^\mu \not{p} \gamma_\mu}_{-2\not{p}} \overset{q}{\gamma^\nu} q_\nu \\
&= 2\gamma^\nu \gamma^\mu p_\mu q_\nu + 2\not{p} \not{q} \\
&= 2(2\eta^{\nu\mu} \mathbb{1} - \gamma^\mu \gamma^\nu) p_\mu q_\nu + 2\not{p} \not{q} \\
&= 4 \cdot \mathbb{1} \underbrace{\eta^{\mu\nu} p_\mu q_\nu}_{p^\nu q_\nu} - 2\not{p} \not{q} + 2\not{p} \not{q} \\
&= \underline{4 p \cdot q \mathbb{1}}
\end{aligned}$$